CS211
Data Structures II

2nd Year B.Sc. Computer Science
Higher Diploma in Information Technology

Course Contents

- Binary trees & ADTs
- Applications of binary trees
  - Huffman codes and compression
- AVL Trees
- Red Black search trees
- Sorting
- Tables and Hashing
- Graphs

Section 1: Binary Trees

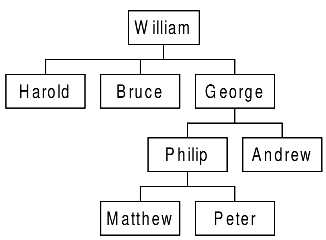
1.1 : Binary Trees, Fundamentals

Binary Trees

- Trees are everywhere
- Hierarchical method of structuring data
- Uses of trees:
  - genealogical tree
  - organisational tree
  - expression tree
  - binary search tree
  - decision tree

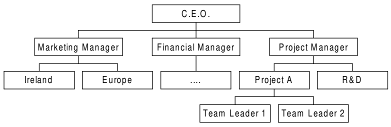
Uses of Trees

Genealogical Tree



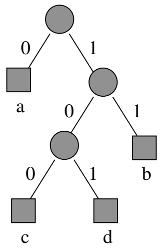
Uses of Trees

Organization Tree



Uses of Trees

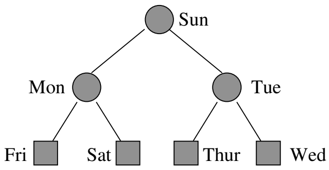
Code Tree



i.e.
d = 101

Uses of Trees

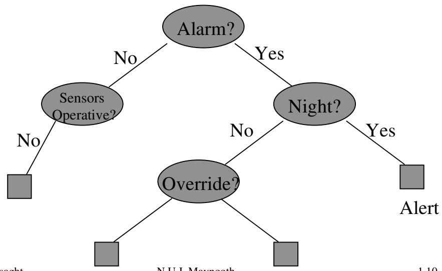
Binary Seach Tree



Ordered sequence of elements stored in a tree structure

Uses of Trees

Decision Tree



Trees

- Fundamentals
- Traversals
- Display
- Representation
- Abstract Data Type (ADT) approach
- Emphasis on binary tree
- Multi-way trees

Tree Definitions

- A binary tree T of n nodes, $n \geq 0$, either is empty, if $n = 0$, or consists of a root node u and two binary trees $u(1)$ and $u(2)$ of n_1 and n_2 nodes, respectively, such that $n = 1 + n_1 + n_2$.
- We say that $u(1)$ is the first or left subtree of T, and $u(2)$ is the second or right subtree of T.

Binary Tree

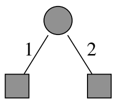
External nodes - have no subtrees

Internal nodes - always have two subtrees

Binary Tree

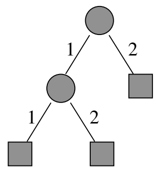
Binary Tree of zero
(internal) nodes

Binary Tree



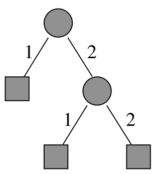
Binary Tree of
one(internal) node

Binary Tree



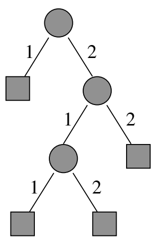
Binary Tree of two
(internal) nodes

Binary Tree



Binary Tree of two
(internal) nodes

Binary Tree

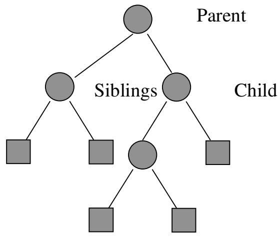


Binary Tree of three (internal)
nodes

Binary Tree Terminology

- Let T be a binary tree with root u
- Let v be any node in T
- If v is the root of either $u(1)$ or $u(2)$, then we say u is the parent of v,
  and that v is the child of u
- If w is also a child of u, and w is distinct from v, we say that v and w are siblings.

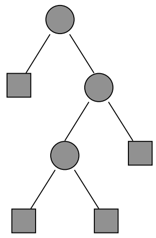
Binary Tree



Binary Tree Terminology

- If v is the root of $u(i)$,
- then v is the ith child of u; $u(1)$ is the left child and $u(2)$ is the right child.
- Also have grandparents and grandchildren

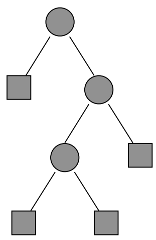
Binary Tree



Binary Tree Terminology

- Given a binary tree T of n nodes, $n \geq 0$,
- then v is a descendant of u if either
  - v is equal to u
    or
  - v is a child of some node w and w is a descendant of u.
- We write $v\ desc_T\ u$.
- v is a proper descendant of u if v is a descendant of u and $v \neq u$.

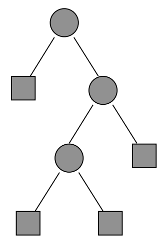
Binary Tree



Binary Tree Terminology

- Given a binary tree T of n nodes, $n \geq 0$,
- then *v* is a left descendant of *u* if either
  - *v* is equal to *u*
    or
  - *v* is a left child of some node *w* and *w* is a left descendant of *u*.
- We write $v\ ldesc_T\ u$.
- Similarly we have $v\ rdesc_T\ u$.

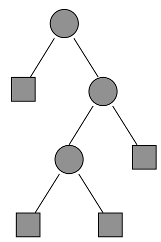
Binary Tree



Binary Tree Terminology

- $ldesc_T$ relates nodes across a binary tree rather than up and down a binary tree.
- Given two nodes *u* and *v* in a binary tree *T*, we say that *v* is to the left of *u* if there is a new node *w* in *T* such that *v* is a left descendant of *w* and *u* is a right descendant of *w*.
- We denote this relation by $left_T$ and write $v\ left_T\ u$.

Binary Tree



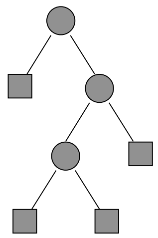
Binary Tree Terminology

- The external nodes of a tree define its frontier
- We can count the number of nodes in a binary tree in three ways:
  - Number of internal nodes
  - Number of external nodes
  - Number of internal and external nodes
- The number of internal nodes is the size of the tree

Binary Tree Terminology

- Let T be a binary tree of size n , $n \geq 0$,
- Then, the number of external nodes of T is $n + 1$

Binary Tree



Binary Tree Terminology

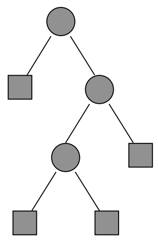
- The height of T is defined recursively as

  0 if T is empty and

  $1 + max(height(T_1), height(T_2))$ otherwise, where T_1 and T_2 are the subtrees of the root.

- The height of a tree is the length of a longest chain of descendants

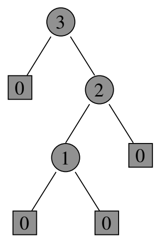
Binary Tree



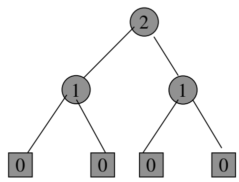
Binary Tree Terminology

- Height Numbering
  – Number all external nodes 0
  – Number each internal node to be one more than the maximum of the numbers of its children
  – Then the number of the root is the height of T
- The height of a node u in T is the height of the subtree rooted at u

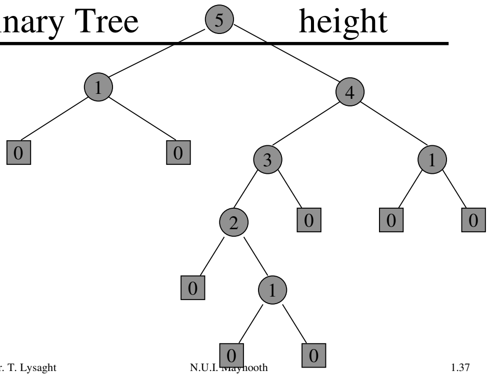
Binary Tree height



Binary Tree height



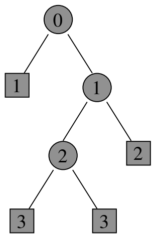
Binary Tree height



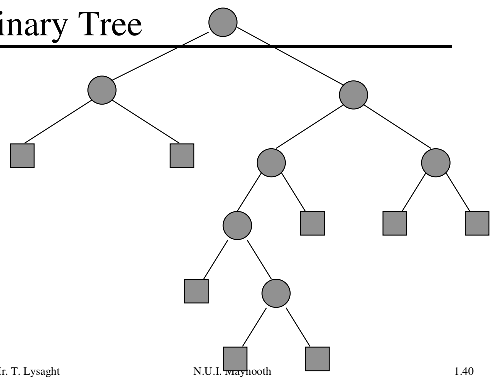
Binary Tree Terminology

- Levels of nodes
  - The level of a node in a binary tree is computed as follows
  - Number the root node 0
  - Number every other node to be 1 more than its parent
  - Then the number of a node *v* is that node's level
  - The level of *v* is the number of branches on the path from the root to *v*

Binary Tree



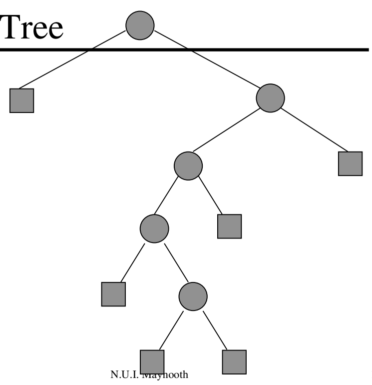
Binary Tree



Binary Tree Terminology

- Skinny Trees
  - every internal node has at most one internal child

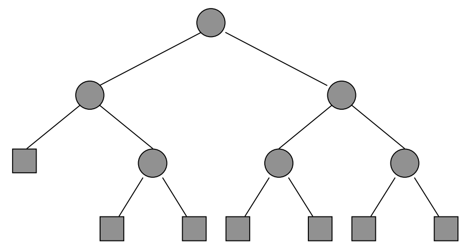
Skinny Tree



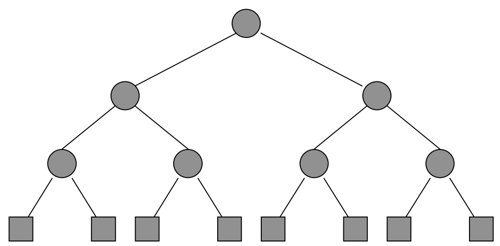
Binary Tree Terminology

- Complete Binary Trees (Fat Trees)
  - the external nodes appear on at most two adjacent levels
  - Perfect Trees: complete trees having all their external nodes on one level
  - Left-complete Trees: the internal nodes on the lowest level are in the leftmost possible position.
  - Skinny trees are the highest possible trees
  - Complete trees are the lowest possible trees

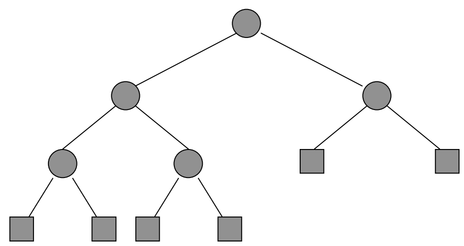
Complete Tree



Perfect Tree



Left-Complete Tree



Binary Tree Terminology

- A binary tree of height $h \geq 0$ has size at least h
- A binary tree of height at most $h \geq 0$ has size at most $2^h - 1$
- A binary tree of size $n \geq 0$ has height at most n
- A binary tree of size $n \geq 0$ has height at least $\lceil \log_2 (n + 1) \rceil$

Multiway Trees

- Multiway trees are defined in a similar way to binary trees, except that the degree (the maximum number of children) is no longer restricted to the value 2

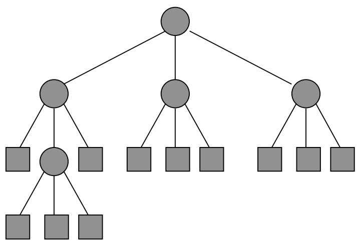
Multiway Trees

- A multiway tree T of n internal nodes, $n \geq 0$,
  - either is empty, if $n = 0$,
  - or consists of
    - » a root node u,
    - » an integer $d_u \geq 1$, the degree of u,
    - » and multiway trees $u(1)$ of n_1 nodes, ..., $u(d_u)$ of n_{d_u} nodes such that $n = 1 + n_1 + \ldots + n_{d_u}$

Multiway Trees

- A multiway tree T is a d-ary tree, for some $d > 0$, if $d_u = d$, for all internal nodes u in T

d-ary Tree



Multiway Trees

- A multiway tree T is an (a, b)-tree, if $1 \leq a \leq d_u \leq b$, for all u in T
- Every binary tree is a (2, 2)-tree, and vice versa